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IDENTIFICATION OF LOCAL THERMAL CONTACT RESISTANCES BY SOLVING THE INVERSE HEAT-CONDUCTION PROBLEMS

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Ways for determination of the local boundary conditions of the fourth kind are outlined. A version of the analog solution of the corresponding inverse problem by the method of spectral functions of influence is given. Results of solving the methodical problem are reported.

In investigating the thermal state of structures, it is often insufficient to know the averaged boundary conditions of heat transfer obtained by empirical relations available in the literature. In the general case, the heat actions at a boundary change not only in time but in space as well. Therefore it is very important to be able to find local boundary conditions that to a considerable degree increase the reliability of information obtained through mathematical modeling. All this pertains in full measure to the boundary conditions of the fourth kind, identification of which has its specific features and is distinguished by the complexity of determination of thermal contact resistances (TCR) by results of thermophysical experiment. As a rule, this involves the entire arsenal of methods and tools employed in solving the inverse heat-conduction problems, including the digital, analog, and hybrid ways of implementation of the processes of modeling and identification of thermophysical processes [1–5].

As for the point-by-point identification of TCR (a one-dimensional problem or a problem in which local or unchanging, over a surface, thermal contact resistances are determined), it can be accomplished on an analog device (Fig. 1) that includes two passive models PM1 and PM2 (for instance, R-grids), on which temperature fields of contacting bodies are modeled, and controllable resistors R1 and R2, the sum of whose resistances is an analog of TCR between bodies, i.e., the analog of TCR represents the electric resistance switched between boundary nodes of the R-grids that model the elements of a composite body. Control of these resistors is carried out by blocks BS1 and BS2 on which the signals corresponding to the temperatures T^{mod} obtained on modeling are compared with signals–analogs of the temperatures T known from experiment (reference points, reference temperatures). Results of the comparison enter the inputs of integrators I1 and I2 at the outputs of which the signals controlling resistors R1 and R2 are formed. The process continues until the signals at the outputs of blocks BS1 and BS2 are equalized to zero, which, in fact, indicates minimization of the functional

$$F_k = \sum_{i=1}^{N} \left| T^{\text{mod}} (x_i, y_i, \tau_k) - T (x_i, y_i, \tau) \right| .$$

Since the problem in the above formulation is an overdetermined one (only one parameter, i.e., TCR and two reference points are identified), the temperature at one reference point is used for determination of TCR while that at the other, for its refinement. In conformity with this, resistors R1 and R2 must have different nominal values (the nominal value of one resistor must exceed by two to three orders of magnitude the nominal value of another).

After completion of control it appears that resistors R1 and R2 are in the state where their total resistance is equivalent to the TCR between the contacting surfaces. A given scheme (Fig. 1) can be considered as the simplest implementation of the principle of determining TCR on analog and hybrid devices.

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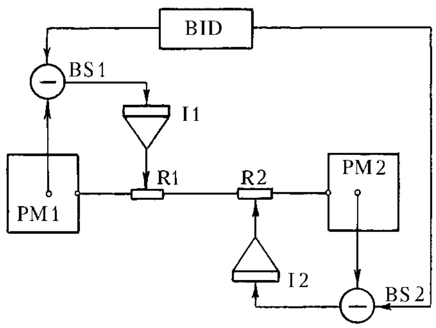

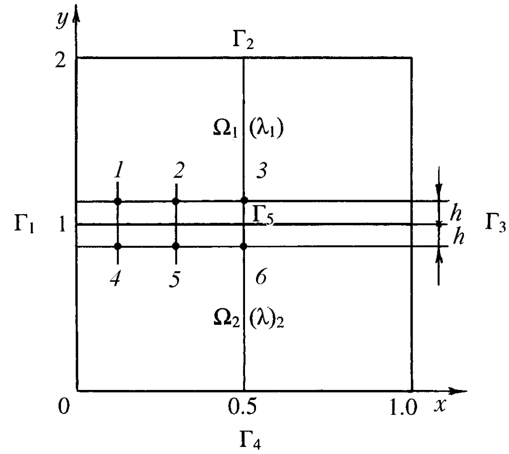

Fig. 1. Implementation of an analog solution of the inverse heat conduction problem on determination of TCR (BID, block of the initial data).

Fig. 2. Schematic representation of the inverse heat-conduction problem.

In actual problems, more complicated situations emerge where, for instance, it is necessary to determine a TCR which is changeable over the boundary and in time but experimental information available for solving such a multiparameter problem is obviously insufficient. In such a case, we are concerned with an undetermined rather than overdetermined problem, and this necessitates the use of more economically attractive ways which allow information on the character of sought dependences to be presented in a more concise form. In particular, for this purpose an approach can be employed based on the use of spectral functions of influence of boundary actions [6, 7] expressing the dependence of the temperature field on its spectral components rather than on the boundary action as a whole.

If we represent functions f_i describing the input boundary actions of the ith section of the boundary of an object in the form

$$f_i = \sum_{j=0}^{m_i} a_{ij}\,\eta_i^j\,; \quad \eta_{1i} \le \eta_i \le \eta_{2i}\,, \quad i = 1, 2, ..., n\,,$$

where the coefficients a_{ij} are the parameters of approximation, then the temperature inside the body can be determined as follows:

$$T = \sum_{i=1}^{n} \sum_{j=1}^{m_i} a_{ij}\,W_{ij}\,, \tag{1}$$

here W_{ij} are the spectral functions of influence. Solution of the inverse heat-conduction problem reduces to solution of the system of nm_i algebraic equations, as a result of which the parameters a_{ij} are determined by the known W_{ij} and the temperature inside the object. The error of their determination depends on the accuracy with which the spectral functions of influence are obtained and on the error of temperature measurements inside the object.

We will consider the procedure of solving the inverse heat-conduction problem on determination of thermal contact resistances, employing, as an example, the contact of two regions having the shape of rectangles with the ratio of sides 2:1 (Fig. 2). The process of heat conduction for regions Ω_1 and Ω_2 is described by the equation

$$\nabla^2 T = 0$$

with the initial temperature distribution

$$T\,(x, y, 0) = \psi\,(x, y)\,,$$

boundary conditions of the first kind at boundaries Γ_i ($i = 1, 2, 3, 4$), and the following condition of nonideal thermal contact of unlike media at boundary Γ_5:

$$\lambda_1 \frac{\partial T_1}{\partial n}\bigg|_{\Gamma_5} = \lambda_2 \frac{\partial T_2}{\partial n}\bigg|_{\Gamma_5}, \quad \lambda \frac{\partial T_1}{\partial n}\bigg|_{\Gamma_5} = \frac{1}{R_c}(T_2 - T_1)\bigg|_{\Gamma_5}.$$

It is necessary to determine the thermal contact resistance R_c at boundary Γ_5 by the known temperatures at the points of observation that are located on both sides of the boundary at a distance of $h/2$.

To calculate TCR, it is necessary to determine the surface temperature of contact $T_{1s}(x, 1)$ and $T_{2s}(x, 1)$ for both regions and the heat flux $q_5(x, 1)$ passing across the boundary of contact and then to calculate the TCR by the formula

$$R_c(x, 1) = \frac{T_{1\mathrm{sur}}(x, 1) - T_{2\mathrm{sur}}(x, 1)}{q_5(x, 1)}. \tag{2}$$

The heat flux $q_5(x, 1)$ is determined in the form of a power polynomial:

$$q_5(x, 1) = \sum_{j=0}^{m_i} a_j x^j, \quad 0 \le x \le 1,$$

where the parameters a_j of the boundary actions are unknown.

Identification of these parameters is carried out by solving a system of algebraic equations:

$$\sum_{j=0}^{m_i} a_j W_j(x_s, 1 + \bar{h})\big|_{\Omega_1} = T_1^k(x_s, 1 + \bar{h}) - T_{\Omega_1}^k(x_s, 1 + \bar{h}), \quad s = 1, ..., N_1; \tag{3}$$

$$\sum_{j=0}^{m_i} a_j W_j(x_s, 1 - \bar{h})\big|_{\Omega_2} = T_2^k(x_s, 1 - \bar{h}) - T_{\Omega_2}^k(x_s, 1 - \bar{h}), \quad s = 1, ..., N_2, \tag{4}$$

where $T_1^k(x_s, 1 + \bar{h})$ and $T_2^k(x_s, 1 - \bar{h})$ are the temperatures at the points of observation in regions Ω_1 and Ω_2 respectively, while $T_{\Omega_1}^k(x_s, 1 + h)$ and $T_{\Omega_2}^k(x_s, 1 - h)$ are the responses to the known boundary actions and the temperature fields at the preceding instant of time $T^{k-1}(x, y)$.

In the case of an overdetermined system of equations, the least-squares method is used, which makes it possible to perform symmetrization of a matrix of the initial system of equations and thus to prepare it for solving.

The surface temperatures $T_{1s}(x, 1)$ and $T_{2s}(x, 1)$ are obtained from a solution of the heat-conduction problem for regions Ω_1 and Ω_2 with the known, from the problem formulation, boundary conditions on surfaces Γ_1–Γ_4 and with the determined heat flux $q_5(x, 1)$; next, the TCR are calculated.

As an example, we solved a methodical problem on determination of the boundary conditions of the fourth kind between regions Ω_1 and Ω_2 with the ratio of thermal conductivities $\lambda_1:\lambda_2 = 1:2$ and nonideal heat contact between them. The necessary "measurements" at the points of observation were taken from a solution of the direct problem. In so doing, thermal contact resistances, both constant ($\bar{R}_c = 1, 3, 5$) and variable ($\bar{R}_c = \bar{R}_{c\,\max}(1 - 3.306x^2)$, where $\bar{R}_{c\,\max} = 1, 3, 5$), were prescribed along the boundary of contacts; $\bar{R}_c = 1$ corresponds to thermal resistance of the layer inside Ω_1 with thickness $\bar{h}$ (where $\bar{h} = 0.1$ is an approximation step).

On the surfaces Γ_1–Γ_4 the boundary conditions of the first kind were set:

$$T(x, y) = 100 \begin{cases} x = 0, & 0 \le y \le 2 \\ x = 1, & 0 \le y \le 2 \end{cases}; \quad T(x, y) = 200, \quad 0 \le x \le 1, \quad y = 1;$$

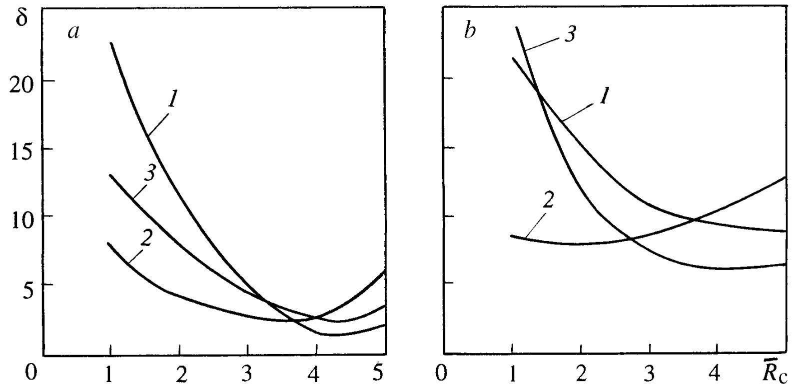

Fig. 3. Maximum root-mean-square deviations of the restored TCR values from the prescribed ones at constant (a) and variable (b) distributions of TCR along the surface of contact.

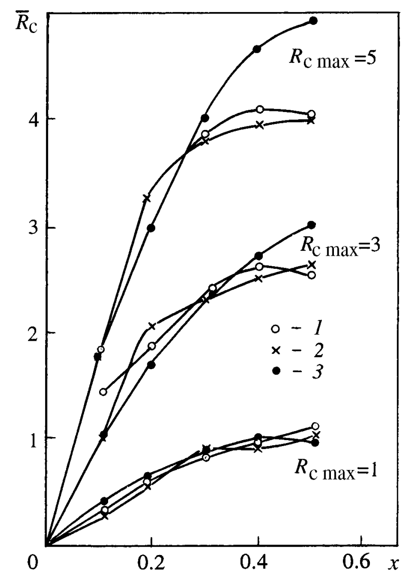

Fig. 4. Comparison of the prescribed and restored TCR values at variable $\bar{R}_c$ along the boundary of contact $\bar{R}_c = \bar{R}_{max}$ $(1 - 3.306x^2)$: 1) prescribed value of TCR; 2) restored value; 3) smoothed value.

$$T(x, y) = 0\,, \quad 0 \le x \le 1\,, \quad y = 0\,.$$

Three variants of installing the temperature pickups were considered:

I, on both sides of boundary Γ_5 (points 1–6);

II, on one side of Γ_5 in region Ω_1 (points 1–3);

III, on one side of Γ_5 in region Ω_2 (points 4–6).

For all three variants, the temperature pickups were installed at a distance of $h/2 = 0.05$ from the boundary of contact.

In the first case, the parameters a_j are determined by solving system (3), in the second and third cases, by solving system (4). The heat flux was approximated by a second-degree polynomial.

Having solved the inverse heat-conduction problem, we determined the root-mean-square deviation δ of the restored values of TCR from the prescribed ones:

$$\delta = \sqrt{\frac{\sum_{i=1}^{5} [\bar{R}_c(x_i) - \bar{R}_c^*(x_i)]^2}{5}} \cdot 100\% .$$

Plots of the dependence of the root-mean-square deviation in determination of $\bar{R}_c$ on the level of TCR for the three variants of location of the temperature pickups are shown in Fig. 3.

Figure 4 gives results of restoration of the TCR which is variable along the surface of contact.

The results reported allow us to draw the conclusion that use of the spectral functions of influence of the boundary actions provides the best results in the case where measurement data are available for one of the contacting regions. Here, the least errors occur if the temperature pickups are placed in the more heated region. Moreover, the general tendency of decreasing the error with an increase in $\bar{R}_c$ has engaged our attention.

The investigations conducted have allowed evaluation of the level of expected methodical errors in determination of TCR. Since solutions of the inverse problems are sensitive to errors of different kinds, including the errors of approximation of the heat-conduction equation, in using the finite-difference method it is necessary to apply a dense grid for the region of contact of the elements of a composite body.

It should be noted that the method of spectral functions of influence employed here for solving the inverse heat-conduction problem on determination of TCR has received further development. The regional spectral functions of influence were suggested [8], which allowed substantial improvement of the stability of solutions obtained with their high accuracy being retained. Subdividing a boundary into regions, one can more roughly approximate the boundary action within the limits of a region, which favors regularization of solution of an ill-posed problem. At the same time, such a "rough" approximation of the boundary actions within the limits of the region virtually does not affect the accuracy of identification of the boundary conditions within the entire boundary subdivided into regions.

NOTATION

T^{mod}, temperature obtained in modeling, K; T, reference temperature, K; F_k, functional of discrepancy; x, y, Cartesian coordinates; τ, time coordinate; f, function describing the input boundary actions; ψ, arbitrary function; η, dimensionless space coordinate; R_c, thermal contact resistance, Ω; q, specific heat flux, W/m^2; s, number of the nodal point in the x-coordinate; λ, thermal conductivity, W/(m·K); h, approximation step, m; δ, root-mean-square deviation, %; R_c^*, prescribed value of TCR, Ω; R_c, restored value of TCR, Ω. Subscripts and superscripts: s, surface; c, contact; Ω, region; max, maximum; k, instant of time.

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